OR Deterministic Qualifying Examination

Information:
- Student’s full name: ________________________________.
- Student’s signature: ________________________________.
- Date: ......./...../2017. Time: ________________________.
- Honor pledge: I have neither given nor received unauthorized assistance on this exam.

General instructions: Please carefully read the following instructions before writing your answers.

a. Write your name on this sheet. Date and sign on the first page.

b. This examination is closed-book, and consists of two questions.

c. Answer all these questions as clearly and concisely as you are able.

d. Use of the internet and/or mobile devices is not permitted.

The exam questions

Question 1: (60 points) We are given two undirected graphs \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \).

a. Assume \( |V_1| = |V_2| = n \) and \( |E_1| = |E_2| = m \). Describe an integer programming (IP) problem, which is feasible if and only if \( G_1 \) and \( G_2 \) are isomorphic, i.e. there exists 
\[ f : V_1 \to V_2 \text{ one to one mapping} \]
such that \( (i, j) \in E_1 \iff (f(i), f(j)) \in E_2 \).

The IP should have 0–1 variables, and a polynomial number of variables and constraints in \( n \) and \( m \).

Carefully explain in words the meaning of the variables and constraints. Especially carefully explain the meaning of the logic of the constraints.

b. For this part, we need the concept of an induced subgraph. If \( G = (V, E) \) is an undirected graph, then the graph \( (W, F) \) is an induced subgraph if \( W \subseteq V, F \subseteq E \) and \( i, j \in E, i, j \in W \) implies \( (i, j) \in F \). That is, if we choose two endpoints of an edge, then we have to choose the edge as well.

Now do not assume that \( G_1 \) and \( G_2 \) have the same number of nodes, or edges. Describe an IP with 0–1 variables to choose a maximum cardinality induced subgraph of \( G_1 \) and of \( G_2 \) which are isomorphic.

The IP again should have a polynomial number of variables and constraints.

Again, carefully explain in words the meaning of the variables and constraints. Especially carefully explain the meaning of the logic of the constraints.
Question 2: (40 points) Consider the following linear programming problem:

minimize \[ 16x_1 + 12x_2 + 10x_3 + 11x_4 \]
subject to \[ 180x_1 + 120x_2 + 90x_3 + 60x_4 - x_5 = 90 \]
\[ 3x_1 + 2x_2 + 6x_3 + 5x_4 + x_6 = 4 \]
\[ x \geq 0 \]

a. Verify that the basis \( \{x_1, x_6\} \) is an optimal basis, and write down the set of all optimal solutions.

b. Let \( c_1 \) be the coefficient of \( x_1 \) in the objective function. Its present value is 16. For what range of values of \( c_1 \) does the basis \( \{x_1, x_6\} \) remain optimal, assuming that all other data entries are fixed at their present values? Moreover, for what values of \( c_1 \) within this range does the problem have multiple primal optimal solutions? Write down the set of primal optimal solutions when \( c_1 \) takes those values.

c. Let \( b_1 \) be the right hand side of the first constraint. Its present value is 90. For what range of values of \( b_1 \) does the basis \( \{x_1, x_6\} \) remain optimal, assuming that all other data entries are fixed at their present values? Moreover, for what values of \( b_1 \) within this range does the dual problem have multiple optimal solutions? Write down the set of dual optimal solutions when \( b_1 \) takes those values.